Increasing return response to changes in risk

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Briefly...

- A positive and marginally diminishing relationship between risk and return exists.
- This study is about investor behavior as it relates to their response (not choice) to risk.
- We present an argument and supporting evidence that investors' return response to risk is increasing in level of risk.

Risk and return tradeoff

- There is no doubt about investors seeking higher compensation for higher risk.
- On a cross section of investment choices, the correlation of risk and return is positive in early evidence.
- More recently, the evidence is mixed for cross-section as well as for time-series analysis.
- Based on a recent sample, 2006-2013, this evidence is in direct disagreement with earlier positive risk-return tradeoff evidence in the literature.

Positive risk and return tradeoff?

Table A-2: Risk-return tradeoff estimation results for the model: $\mu r_i = \alpha + \beta\sigma_i + \epsilon_i$ and $\sum r_i = \alpha + \beta\sigma_i + \epsilon_i$ where $r_{i,t} = ln(P_{i,t}/P_{i,t-1})$, $\mu r_i = \frac{\sum_{t=1}^{N} r_{it}}{N}$, $\Sigma r_i = \sum_{t=1}^{N} r_{it}$ and $\sigma_i = \left(\frac{\sum_{t=1}^{N}(r_{it}-\bar{r}_i)^2}{N-1}\right)^{1/2}$. *, ** and *** refer to statistical significance at the 10%, 5% and 1% respectively.

	μr_i		Σr_i	
σ_i	-0.002		-0.581	
	(0.011)		(2.839)	
α	0.001	***	0.175	***
	(0.000)		(0.043)	
R^2	-0.015		-0.015	
N	65		65	

Risk and return tradeoff

- In terms of bonds, there is no doubt about this positive and statistically significant tradeoff.
- While stock investors have the same risk-return tradeoff, they cannot possibly know all future payments with certainty.
- Ex-ante, investors chose stocks based on their possible (predicted or expected) future risk-return schemes.
- Ex-post, investors realize the risk and returns as they materialize.
- Thus, any evidence about positive and statistically significant risk-return tradeoff, ex-post, is in effect an evidence of investors success in prediction.

Risk and return tradeoff

- Bond prices are present values of all future expected payments.
- If risk increases investors will decrease (by paying less) bond prices to a matching bond yield.
- This is also the case for stocks.
- If stock prices are present values of all future expected payments, much like bonds, then the same yield theory would apply to stocks.
- As risk increases investors would decrease stock prices to a matching yield.

Risk and return tradeoff

- *While overall risk and return tradeoff stays positive for cross section of stocks, ex-ante, daily returns would have negative correlation with daily changes in risk on a time series analysis*.
- We provide evidence that the correlation between daily returns and expected risk (implied volatility) is statistically significant and negative for major US equity indexes, international equity ETFs and a few select stocks.

Daily returns and expected risk (implied volatility)

Table 1: **Estimation results for market returns (ex. $\Delta SP500_t$) and their corresponding implied volatilities (ex. $\Delta IV_{VIX,t}$).** The correlation between daily returns ($r_{i,t}$) and expected volatility ($E(\sigma_{i,t})$) is estimated as follows: $\Delta Security_{it} = \beta_{i0} + \beta_{i1}\Delta IV_{it} + \epsilon_{it}$ and $\sigma^2_{it} = \alpha_{i0} + \sum_{j=1}^{q} \alpha_{ij}\epsilon^2_{i,t-j} + \sum_{j=1}^{p} \gamma_{ij}\sigma^2_{i,t-j}$ where $\epsilon_{it}|\delta_{i,t-1} \sim N(0, \sigma^2_{it})$. *Security* refers to daily closing prices for individual equities, equity indexes and equity ETFs. *IV* refers to implied volatility index corresponding to the underlying equity. Equity data are obtained from Yahoo! Finance. Implied volatility data are obtained from Chicago Board Options Exchange (CBOE). Data period begins January, 2000 and ends September, 2017. *, ** and *** refer to statistical significance at the 10%, 5% and 1% respectively.

$Security_i$	IV_i	β_{i1}		β_{i0}		$Arch_{t-1}$		$Garch_{t-1}$		R^2	χ^2	N
S&P 500	VIX	-0.0966	***	0.0004	***	0.0962	***	0.8930	***	0.5434	12,834.68	4,457
NASDAQ 100	VXN	-0.1287	***	0.0006	***	0.0583	***	0.9369	***	0.3942	7,199.84	4,189
S&P 100	VXO	-0.0851	***	0.0004	***	0.0958	***	0.8983	***	0.5256	12,849.20	4,457
Dow 30	VXD	-0.0932	***	0.0004	***	0.1167	***	0.8745	***	0.4614	11,412.76	4,457
Russell 2000	RVX	-0.1632	***	0.0005	***	0.0798	***	0.9043	***	0.5428	7,742.41	3,450
S&P 500	VXST	-0.0418	***	0.0006	***	0.1368	***	0.8306	***	0.5042	3,845.14	1,689
S&P 500	VXV	-0.1629	***	0.0004	***	0.1285	***	0.8556	***	0.6064	22,551.44	2,465
S&P 500	VXMT	-0.2376	***	0.0004	***	0.1329	***	0.8557	***	0.6521	17,836.71	2,443
iShares MSCI EAFE ETF	VXEFA	-0.1004	***	0.0002		0.1316	***	0.8503	***	0.3805	4,616.20	2,446
iShares MSCI Emerging Markets ETF	VXEEM	-0.1387	***	0.0002		0.1036	***	0.8540	***	0.5278	2,995.72	1,639
iShares China Large-Cap ETF	VXFXI	-0.1690	***	0.0002		0.0372	***	0.9440	***	0.3280	1,162.34	1,639
iShares MSCI Brazil Capped ETF	VXEWZ	-0.2484	***	-0.0002		0.0999	***	0.8847	***	0.3185	2,897.82	1,639
Amazon.com, Inc.	VXAZN	-0.0782	***	0.0013	***	0.2666	***	0.0069		0.0793	1,778.84	1,839
Apple Inc.	VXAPL	-0.1051	***	0.0010	***	0.1253	***	0.7664	***	0.1904	1,280.58	1,839
The Goldman Sachs Group, Inc.	VXGS	-0.1650	***	0.0002		0.0343	***	0.9564	***	0.3939	3,192.17	1,839
Alphabet Inc	VXGOG	-0.0871	***	0.0006	**	0.2034	***	-0.0304		0.1781	1,749.41	1,839
IBM	VXIBM	-0.0653	***	0.0002		0.1764	***	0.0672		0.1160	810.49	1,839

Price response to changes in risk

- We argue that the investors' risk-return behavior changes when risk is above the "usual" level.
- It is established that investors in fact overreact to events
- It is also evident that investors' risk aversion changes based on habits and overall risk levels.
- Guiso (2013) show that investors' risk aversion has increased after the 2008 financial crisis.
- Effect of fear (fear of financial crash) on further choices made by investors.

Price response to changes in risk

- Behavioral sciences offer evidence to this effect with respect to behavior for health risks.
- Brewer (2004) argues that as individuals' perception of higher risk leads them to seek protection (i.e. vaccinations).
- This evidence is supported by Holt (2002) who show that the pay-out has a direct impact on the risk choices of individuals.
- We argue that investors would be expected to change their response to risk when faced with fear.

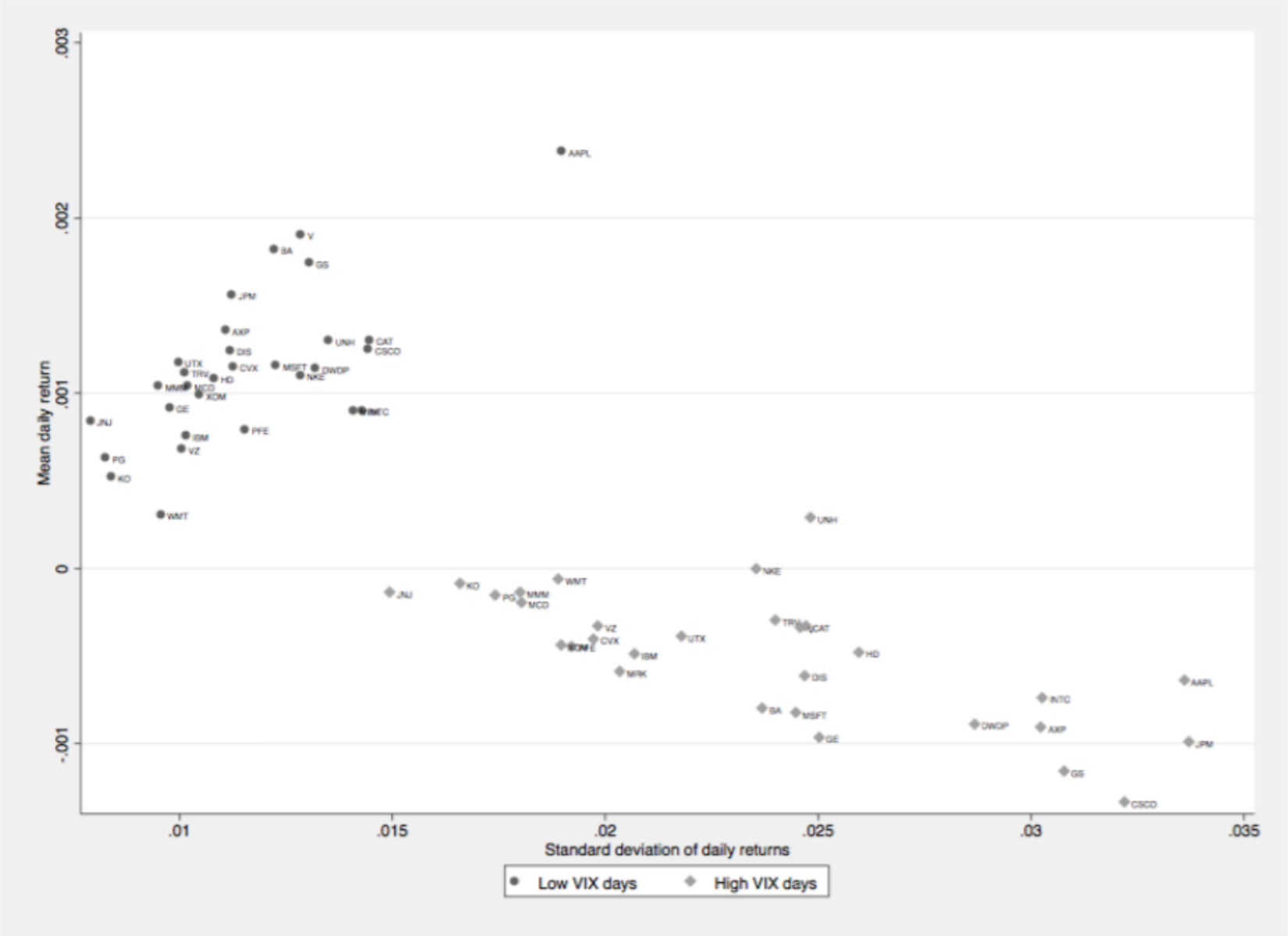
Price response to changes in risk

- Implied volatility indexes for the equity markets (i.e. VIX) is considered to be the fear gauge.
- A long run average of the VIX would create a feeling of normality (almost like habit formation).
- Levels of risk above this long run average would leave investors feeling vulnerable (i.e. in fear).
- When in fear investors would be expected to react less rationally, more quickly and perhaps in exaggeration.
- We therefore argue that *investors' price response to increases in risk would be higher for fear periods*.

Price response to changes in risk

- Given the negative time-series return response to changes in risk, we would expect this negative response to be even more for days when VIX is above its long run average.
- Our evidence supports this theory.
- Equities have completely different risk-return tradeoff cross-sectionally for high risk days and for low risk days.
- They also have significantly lower negative return response to changes in risk for high risk days compared to low risk days.

Price response to changes in risk



Price response to changes in risk

$$\Delta Index_{it} = \beta_{i0} + \beta_{i1,low}\Delta IV_{low,it} + \beta_{i2,high}\Delta IV_{high,it} + \epsilon_{it} \tag{5}$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{j=1}^{q} \alpha_{ij}\epsilon_{i,t-j}^2 + \sum_{j=1}^{p} \gamma_{ij}\sigma_{i,t-j}^2 \tag{6}$$

where

$$\epsilon_{it}|\delta_{i,t-1} \sim N(0, \sigma_{it}^2) \tag{7}$$

$$\Delta IV_{\mathbf{low},it} \begin{cases} \Delta IV_{it}, & \text{if } IV_{it} < \widetilde{\mu}_{IV_i} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

$$\Delta IV_{\mathbf{high},it} \begin{cases} \Delta IV_{it}, & \text{if } IV_{it} \geq \widetilde{\mu}_{IV_i} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Price response to changes in risk

Table 6: **Estimation results for market returns (ex. $\Delta SP500_t$) and their corresponding implied volatilities (ex. $\Delta IV_{VIX,t}$) for low risk days and high risk days.** Long-run median of VIX ($\widetilde{\mu}_{VIX}$) is used to differentiate low risk days ($VIX_t < \widetilde{\mu}_{VIX}$) from high risk days ($VIX_t >= \widetilde{\mu}_{VIX}$). Equations 5 and 6 are estimated as follows: $\Delta Index_{it} = \beta_{i0} + \beta_{i1,low}\Delta IV_{low,it} + \beta_{i2,high}\Delta IV_{high,it} + \epsilon_{it}$ and $\sigma^2_{it} = \alpha_{i0} + \sum_{j=1}^{q} \alpha_{ij}\epsilon^2_{i,t-j} + \sum_{j=1}^{p} \gamma_{ij}\sigma^2_{i,t-j}$ where $\epsilon_{it}|\delta_{i,t-1} \sim N(0, \sigma^2_{it})$. Equity data are obtained from Yahoo! Finance. Implied volatility data are obtained from Chicago Board Options Exchange (CBOE). Data period begins January, 2000 and ends September, 2017. *, ** and *** refer to statistical significance at the 10%, 5% and 1% respectively.

Security	IV	$\Delta IV_{low,t}$	$\Delta IV_{high,t}$	Constant	$Arch_{t-1}$	$Garch_{t-1}$	R^2	χ^2	N
Δ S&P 500	ΔVIX_t	-0.0816 ***	-0.1218 ***	0.0005 ***	0.0988 ***	0.8869 ***	0.5867	21,368.29	4,457
Δ NASDAQ 100	ΔVXN_t	-0.1056 ***	-0.1695 ***	0.0008 ***	0.0555 ***	0.9400 ***	0.4342	10,443.46	4,189
Δ S&P 100	ΔVXO_t	-0.0702 ***	-0.1105 ***	0.0005 ***	0.0957 ***	0.8927 ***	0.5866	20,484.32	4,457
Δ Dow 30	ΔVXD_t	-0.0776 ***	-0.1133 ***	0.0005 ***	0.1218 ***	0.8682 ***	0.4993	18,699.92	4,457
Δ Russell 2000	ΔRVX_t	-0.1409 ***	-0.1957 ***	0.0006 ***	0.0840 ***	0.8969 ***	0.5697	9,518.04	3,450
Δ S&P 500	$\Delta VXST_t$	-0.0332 ***	-0.0501 ***	0.0008 ***	0.1256 ***	0.8357 ***	0.5356	4,562.45	1,689
Δ S&P 500	ΔVXV_t	-0.1391 ***	-0.2058 ***	0.0005 ***	0.1290 ***	0.8485 ***	0.6448	14,584.03	2,465
Δ S&P 500	$\Delta VXMT_t$	-0.2120 ***	-0.2724 ***	0.0004 ***	0.1222 ***	0.8617 ***	0.6781	16,633.70	2,443
Δ iShares MSCI EAFE ETF	$\Delta VXEFA_t$	-0.0695 ***	-0.1478 ***	0.0005 ***	0.1326 ***	0.8495 ***	0.4445	6,040.93	2,446
Δ iShares MSCI Emerging Markets ETF	$\Delta VXEEM_t$	-0.0981 ***	-0.1830 ***	0.0004 **	0.1100 ***	0.8566 ***	0.5663	7,000.25	1,639
Δ iShares China Large-Cap ETF	$\Delta VXFXI_t$	-0.0993 ***	-0.2167 ***	0.0005	0.0360 ***	0.9530 ***	0.3585	1,567.69	1,639
Δ iShares MSCI Brazil Capped ETF	$\Delta VXEWZ_t$	-0.2022 ***	-0.2999 ***	0.0000	0.1115 ***	0.8723 ***	0.3155	3,556.13	1,639
Δ Amazon.com, Inc.	$\Delta VXAZN_t$	-0.0445 ***	-0.1532 ***	0.0017 ***	0.3711 ***	0.0004	0.1034	2,318.54	1,839
Δ Apple Inc.	$\Delta VXAPL_t$	-0.0681 ***	-0.1535 ***	0.0011 ***	0.1599 ***	0.7104 ***	0.2162	1,445.25	1,839
Δ The Goldman Sachs Group, Inc.	$\Delta VXGS_t$	-0.1292 ***	-0.1947 ***	0.0004	0.0524 ***	0.9327 ***	0.4071	3,762.23	1,839
Δ Alphabet Inc	$\Delta VXGOG_t$	-0.0545 ***	-0.1533 ***	0.0014	0.2174	-0.0473	0.2176	4,560.31	1,839
Δ IBM	$\Delta VXIBM_t$	-0.0243 ***	-0.1159 ***	0.0006 **	0.1404 ***	0.0228	0.1735	1,581.56	1,839

Implications

- *Risk-return tradeoff is different for ex-ante and ex-post*.
- *While holding period risk-return tradeoff is positive (ex-ante), stocks' price response to changes in risk is expected to be negatively correlated, much like bonds*.
- This yield theory for stocks would explain the conflicting and mixed results in the literature.

Implications

- *Investors' mode of response changes with higher than usual risk levels.*
- It is similar to driving.
- At higher speeds (i.e. above the usual speed limit), we become more alert, more ready to react and our response would increase.
- Investors' reaction to increases in risk is the same.
- At high levels of risk (i.e. above the usual levels), we become more responsive.